The edge of time: can Graph Theory be used to analyse relationships in B2B networks?

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Abstract.

In previous research it was proposed that parallels existed between the concepts and ideas found in the Science of Networks (SON) literature (Barabasi, 2002; Albert et al, 1999; Buchanan, 2002) and IMP literature (Sutton-Brady and Donnan, 2003). Having identified parallels the challenge now is to investigate whether Graph Theory, the fundamental underpinning of SON, can be using in analysing networks and relationships.

Popularly graphs are perceived as typically involving an X and Y axis, however unlike these continuous graphs, graphs as defined by Euler are “finite in structure and can be used to analyse relationships” (Grimaldi, 1994). Using Euler’s definition of Graph Theory this paper will investigate the concept of connections in B2B networks.

For the purposes of this research study the connection between business entities, or in terms of Graph Theory the Edge, will be investigated longitudinally. Using a selection of Australian companies, this longitudinal study involves examining the connections between them and certain service providers. Essentially this study attempts to use Graph Theory to simplify the depiction of networks over time.

Introduction

In previous research it was proposed that parallels existed between the concepts and ideas found in the Science of Networks (SON) literature (Barabasi, 2002; Albert et al, 1999; Buchanan, 2002) and IMP literature (Sutton-Brady and Donnan, 2003). Having identified parallels the challenge now is to investigate whether Graph Theory, the fundamental underpinning of SON, can be using in analysing networks and relationships.

The importance of attempting to find a means of simplifying the study of networks that could be used by all IMP researchers cannot be understated. Leek et al (2001) stated that there were "significant problems regarding the definition and validity of B2B terms and methodology". They cited the major issue as being the problem of measuring concepts in B2B research and in fact went on to state that "Standard measurements for various concepts and terms would enable research to develop in a more unified manner".

IMP researchers have long been fascinated with network structures. The drawing of complex "ball and stick" models features still in IMP publications. Easton et al, (1997) posit that the intricate ball and stick model "opened a new and exciting world to those more used to the atomistic, homogenous world of traditional research in marketing". The original IMP2 research instrument featured a "diagram" of such a generalised model. This model has some similarity with a branch of mathematics known as graph theory – a graph (or network) being a bunch of nodes connected by links (Barabasi, 2002). Barabasi (2002) describes how Erdos and Renyi proposed the explanation that nodes are connected randomly. However this model's fundamental tenet that all nodes are created equal is not reflected in IMP research reality. Random graph theory seems to have little to offer in explaining the directed nature of connections in business networks. As Easton *et al* (1997) stated, "…what they (structural network studies) fail to do is to go below the surface and attempt to research and/or model the mechanism, deep structures and fundamental processes that drive such (evolutionary) changes. In his seminal work on evolutionary micro-economics Potts (2000) asserts that "Change happens, it happens broadly and deeply, and we require a framework for analysis of its economic aspects…..One thing is apparent though our standard tools of economic analysis are doing a rather poor job explicating what is occurring."

For IMP researchers the connections between entities are generally, but not exclusively, defined in terms of economic exchange relationships. An extreme view is that by tracing economic exchange links it could be shown that any organisation is linked with any other. "In reality, the market is nothing but a directed network. Companies, firms, corporations, financial institutions, governments, and all potential economic players are the nodes." (Barabasi, 2002). Most IMP researchers have concentrated on dyads, focal groups or small nets as the research unit. However the very essence of the network is its connections which the narrow focus on small units does not capture. This idea is consistent with Potts (2000) where he states that it is the connections that are changing and calls for a formal framework that makes the connections the prime variables.

This paper aims to investigate whether the graph theory so ubiquitous in the science of networks theory can be utilized to better understand business networks by simplifying their depiction over time. For the purposes of this research study the connection between business entities, or the edge, will be investigated longitudinally. This research mirrors to a certain extent the technique employed by Holland (1994) but builds on the use of the methodology by applying it longitudinally. Using a selection of Australian companies, this longitudinal study involves examining the connections between them and certain service providers. We will therefore start with a discussion of connections in networks followed by an explanation of graph theory, this theoretical underpinning will set the scene for the research which we have undertaken.

Connections in business networks and the edge

"We are each nested in a cacophony of relations with other actors in society. These relations serve to define our existence in society" (Burt, 1976). Relationships in business markets have come to understood more and more within such a network context. The rationale behind this is clearly that the relationship cannot be viewed as being created or developed in isolation. Potts (2000) posits that economic reality is a dance of connections forming and reforming and by this process, structuring and restructuring the nature of the economic system. Additionally the relationship is seen as embedded in or connected to other relationships. "Generalised connectedness of business relationships implies the existence of an aggregate structure, or form of organisation, we have chosen to qualify as a network" (Hakansson & Snehota, 1995). Given this connectedness it is difficult to define and delimit the boundaries of the network that surrounds any one company (Ford et al, 2003). Research in networks is as a result a complex and indeed complicated task. Companies within the network are seen as nodes in "an ever changing pattern of interactions" (Ritter, 2000). Therefore the connection between the nodes is technically the interactions that take place in the exchange relationship.

According to Freeman (1979) "when two points are directly connected by an edge they are adjacent", from this concept of basic graph theory we can define that connection between two entities in a network is an edge. The aim is to examine the connections/edge within the networks studied and given that researchers who formed the original IMP group advocated the idea that relationships are long-term in nature (Ford, 1997), it is felt that utilising duration makes the most sense. The duration of the connection which exists may be seen as leading to closeness. As an aside, closeness in business relationships often considered a positive variable may have far reaching negative implications. Closeness as we have seen in recent times can have a downside as was evident to a certain degree in the Enron downfall, where some commentators felt they were too close to their auditors. In a similar vein the National Australia Bank (NAB) recently dumped its' auditor KPMG after a series of financial disasters (Financial Review, 2004), most noteworthy here is the fact that the auditor had been serving the bank for 85 years. What reporters have referred to as restructuring key areas of the bank could also be seen as an attempt to change their position in the network. Ford et al (2003) comment on companies facing important choices between accepting their current position as defined by existing relationships or changing their position through new relationships. NAB by dumping their auditor may be seeking to reposition themselves in the network and change the bad perceptions currently in the marketplace which has seen their share of commercial companies drop from 18.9% in August 2003 to 17.8% in February 2004 (Fin Review, 2004a).”Mangers must observe market changes and never hesitate to question long held assumptions and beliefs and eventually terminate the relationship” (Li & Ng, 2002).

Anderson et al (2003) reassert the view that a key issue of network development is change, which they specifically relate to “connectedness in time”. They state that dynamics are ever present in business networks. The word dynamic has its very context in time therefore looking at connections between the entities in the network by its nature has to be time or duration driven. This notion is reaffirmed by earlier work by Easton and Araujo (1994) where in their discussion of networks they refer the fact that “network change has to be observed against a background of stability”. Additionally Easton (1993) states that “stability is a function of the history of a relationship”. Hence the emphasis in this study on a longitudinal approach.

The issue of complexity in business networks must also be noted, since it believed to complicate any study of networks. Economic systems have long been described as complex. While the meaning of complexity is often discussed in terms of order and disorder, Potts (2000) refers to the complexity of the economic system in relation to its structural composition. He says that the specific state of the system can be determined by the connections that thread the elements together. Networks by their nature are dynamic and complex which has lead some researchers to investigate ways of managing complexity in networks (Langlois, 2002; Brusconi and Prencipe, 2001). Langlois (2002) in particular introduces modularity as “a very general set of principles for managing complexity”. His idea is that by breaking up a complex system into discrete pieces the “unmanageable spaghetti tangle of systemic interconnections” is eliminated. However the use of modularity in networks should be approached very carefully since modularity could be seen as reductionist and as Kauffman (1995) warns using a reductionist approach that is breaking complex systems into simple parts while spectacularly successful can leave a vacuum. He highlights the difficulty lying in the fact that “the complex whole may exhibit properties that are not readily explained by understanding the parts.”

In researching the connections between the business entities and despite advice from Ford et al, (2003) that a company-centred view of the network provides an inadequate basis for understanding the dynamics within that world, we plan to proceed to test whether graph theory has a place in network theory using focal companies. It should be noted at this point that we are attempting to understand just one small part of the overall network and further research will be needed to see if the use of graph theory can be expanded past this point.

Graph Theory

Earlier research (Sutton-Brady & Donnan, 2003) observed that the “ball & stick” diagrams so favoured by IMP researchers to depict “networks” of commercial entities had some similarities with a branch of mathematics known as Graph Theory. Unlike many areas of mathematics the theory of graphs has a definite starting point - the 1736 paper Euler published on the seven bridges of Konigsberg problem. The word "graph" here does not refer to the continuous graph of a function such as $y = ax^2 + bx + c$. The "graphs" here are finite in structure and have two ingredients: a set of vertices or "nodes" and a set of "edges" that connect them. A map of roads connecting a number of cities can be represented by a diagram called a graph (or network) where the lines (or roads) are the "edges" and the points (or cities) are referred to as "nodes". Hence we are dealing with two distinct sets of objects: towns (nodes) and roads (edges). Such sets of objects can be used to define a relation. These relationships can be analysed in many different settings. Consider the "ball & stick" model (or graph) of the simplest of business networks, as shown in Figure 1. In the business network shown the nodes represent the business entities and the edges are the connections between the entities. In social network analysis the "edges" are given values which reflect the intensity or strength of that relationship (Loosemore, 1998).

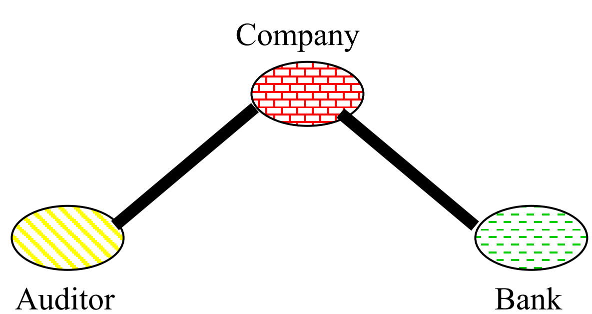

Figure 1: A simple business network.

Potts (2000) is an ardent advocate for the adoption of graph theory in microeconomics "Graph theory is a powerful and obvious foundation for evolutionary economics". Although fundamentally representing economic exchanges B2B connections are multi dimensional. This very multi faceted characteristic at this point inhibits the adoption of much of graph theory for the analysis of business network. When, for example, applied to road maps the edges can "measured" as the physical distance between nodes. Holland (1994) suggests that "The use of matrices to represent market structures is very powerful because it enables the use of simple matrix operations to glean details about market network structure very easily and it also allows the use of sophisticated mathematical techniques to assess structural and graph properties of market networks using matrix based algebra techniques." However the length of the edges, between business entities, defies measurement in simple arithmetic terms. Even in the simplest case the edges minimally represent the economic exchange between the business entities. However this exchange is in reality a series of discrete transactions of varying size and frequency. Add to this the other dimensions of technology, social and contractual arrangements to the edge and edge quantification is a little more daunting than the length of a road between two points.

Perhaps like the difficulty in econometrics, measurement in a dynamic environment "represents the search for the celestial mechanism of a non-existence universe" (Boulding, 1991). Boulding goes on to suggest that the need is for "the development of a mathematics which is suitable to social systems….. We need non-Cartesian algebra as we need non-Euclidean geometry, where minus minus is not always plus, and where the bottom line is often an illusion".

Another of the characteristics of business graphs (or networks) is their longevity. It is in the physical depiction of this characteristic of business networks that we suggest graph theory may have more immediacy. A matrix can be used to store information about a network. Consider the simple "graph" of one edge connecting two nodes O_A to $O_{B,}$ this graph can be written as a matrix as depicted in Figure 2.

$\mathbf{0_A}$**----------------------**$\mathbf{0_B}$

	A	B
A	0	1
B	1	0

Figure 2: Matrix representation of Graph

The matrix corresponding to a graph is defined as its adjacency matrix. An adjacency matrix is a numerical representation of the relational data contained within the graph (network).The graph (network) in Figure 1 can be written as a matrix.

	Company	Auditor	Bank
Company	0	1	1
Auditor	1	0	1
Bank	1	1	0

Figure 3: Matrix depiction of business network

The numerals in the matrix indicate the number of edges connecting each node. In applications such as road maps the distance between nodes (cities) can be stored and the matrix is described as a distance matrix. A scale diagram of the road network can be reconstructed from such a matrix. As stated earlier, measurement of "edges" in B2B networks is problematic.

Research/methodology

Using a selection of Australian companies, this longitudinal study involves investigation of connections between them and certain of their service providers. A longitudinal study was considered necessary to identify and record the changes in the network connections and pattern of relationships over time. The focus on service providers was based on the fact that many researchers have identified services in business markets as an area where research needs to be focussed, given the changing nature of industry worldwide (Tyler, 1996, Leek et al, 2001). Indeed Leek et al, (2001) highlight the fact that in the last twenty years there has been a decline in manufacturing and a huge growth in the services sector, especially in the developed economies.

It should be noted at this point that the Australian economy is highly concentrated and characterised by monopolies or duopolies in key sectors. In such economies the 80/20 Pareto Principle tends toward 95/5. While this may limit the ability to project the nature of these networks to other economies, it does provide a basis for identifying the longevity or stability of networks in this economy.

The companies chosen were a selective sample based on the public availability of data. In total 20 companies (See table 1) from various industries were investigated, the data collected for a ten year period identified the company's banker and auditor. The collection of data was via a commercial database **Connect4** which provides Australian Company Information. Services include Annual Reports, Boardroom (senior salary packages), Company Prospectuses, Mergers & Acquisitions, New Issues, and Takeovers. (http://0-www.connect4.com.au.opac.library.usyd.edu.au/products/ar/index.html)

By employing the Connect4 data base only the organisational relationships for the top 500 Australian Stock Exchange (ASX) listed companies were available for analysis. Moreover these publicly held firms tend to be large with relatively easily identifiable supplier relationships. One might argue that there is a great deal more data available that is not as readily discernable. We selected this database for precisely the reason that it would enable the illustration of the longevity of relationships, albeit very select, for such firms. For standardisation purposes companies are identified using their ASX code.

Table 1: Selected Sample of ASX listed companies.

Company	ASX code	Industry
Adelaide Brighton Cement Holdings Ltd	ABC	Cement
Alcoa	AAI	Aluminium
Amcor Limited	AMC	Packaging
Billabong	BBG	Clothing
Boral Ltd	BOR	Energy
Bridgestone	BDS	Tyre
Coca Cola Amatil	CCL	Beverage
Cochlear	COH	Medical
Cockatoo Ridge Wines	CKR	Wine
Coles Myer Limited	CML	Retailing
CSR Limited	CSR	Sugar
Evans & Tate	ETW	Wine
Fosters	FGL	Brewing
Gowing Bros Ltd	GOW	Retailing
John Fairfax Holdings Limited	FXJ	Publishing
Peter Lehmann Wines	PLW	Wine
McGuigan Simeon Wines	MGW	Wine
Qantas	QAN	Air Transport
Rio-Tinto	RIO	Mining
Singapore Tel	SGT	Telecommunications
Southcorp	SRP	Wine
Xanadu Winery	XAN	Wine

Outcomes

In this study it was found that 80% of companies continue to procure the services of the same bank and the same auditor over the 10 year period. Stability is a characteristic of the majority of these cases in the provision of banking and auditing services. This outcome is consistent with that evidenced by many IMP researchers.

This stability over time would have been traditionally depicted in a “ball and stick model” as seen in Figure 4. However it is clear that while this is easy to do when we are only looking at 3 entities, to represent a more complicated network over time would be near to impossible. In reality such a generalised depiction would just be too cumbersome to construct.

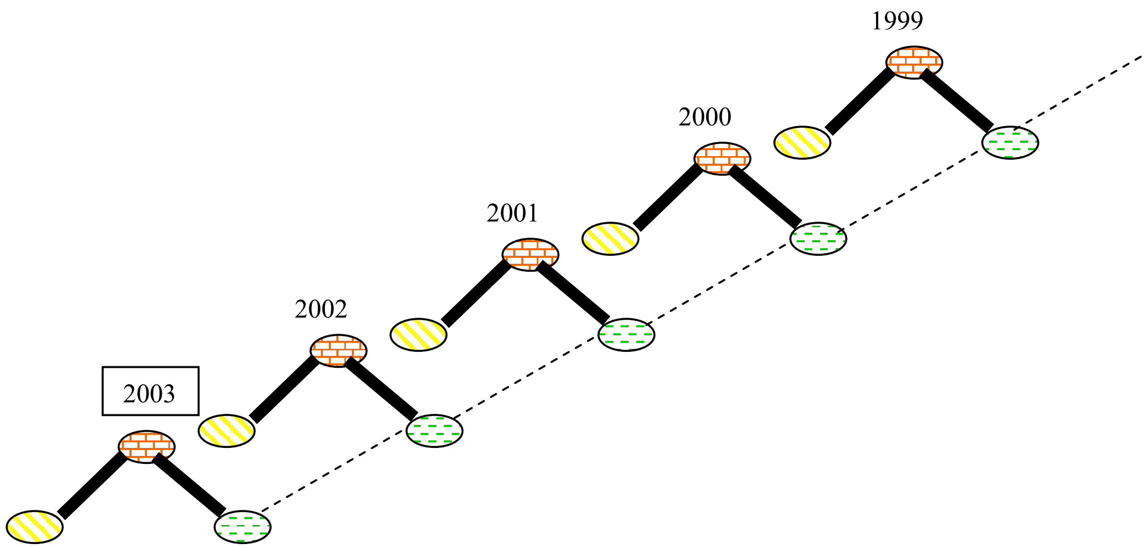

Figure 4: Series of Networks (graphs) over time

Essentially what we are reaffirming is that while depiction of such a series of “graphs” does illustrate the unchanging nature of a network over a limited time it would be impractical for depiction of full and detailed networks over much more than two time periods. Identifying the nature and timing of any network changes would also be difficult. We consider that depicting graphs with adjacent matrices overcomes this difficulty. For example the graph (network) for one of the companies researched: Southcorp Limited.

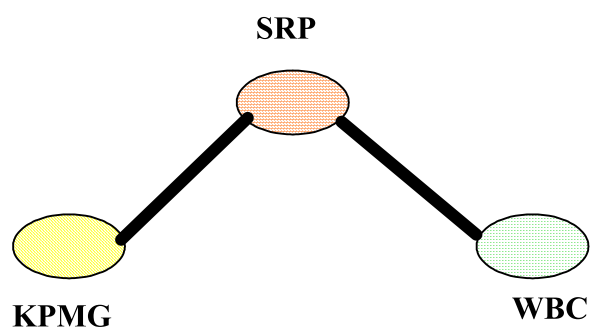

Figure 5: Southcorp Ltd. Graph

This graph can be converted very simply to its' adjacent matrix.

	SRP	KPMG	WBC	
SRP	0	1	1	
KPMG	1	0	1	
WBC	1	1	0	n=30

Figure 6: Matrix of Southcorp Ltd. depicting duration

Given that the matrix is a depiction only – it does not have the algebraic properties of matrices – a further refinement would be to indicate the longevity of this network with a subscript ***n***, where ***n*** = the duration of the network in units of time. For a network the adjacent matrix is simple to construct, record and the storage requirement is minimal. The complexity of graphed network structures has long been an inhibitor of recording and depicting such data. The outcome here suggests that conversion to the adjacent matrix removes the inhibition and simplifies the depiction of networks.

Conclusions

The aim of this paper was to illustrate the value of depicting networks as matrices. It was never intended that the properties of matrices be employed as a way to analyse these networks although initially we did think this may be possible. The depiction of graphs (networks) as matrices does invoke some advantages. Connection changes, and their timing, can more easily be identified from the matrix subscript and storage of network data over time is simplified.

From a managerial point of view the importance of depiction of B2B networks as matrices is twofold: firstly it emphasises market complexity and secondly identifies "nodes" and their "edges". Additionally using matrices to store longitudinal information about networks provides management with some advantages: (1) file space efficient storage of network data, (2) easy reconstruction of the network "graph", and (3) easy computer enabled identification of network changes.

However a limitation of the use of graph theory in business networks is that the connections are not metric. Unlike the length of road between two points the length of a connection between two companies cannot be expressed in simple arithmetic real number terms. At this time there is no "standard of measure" of such a multi-dimensional "connection". This largely negates the adoption of matrix algebra for network analysis purposes thus reducing the role of matrix algebra here to one of depiction and storage only.

This paper clearly opens the way for future research into unearthing tools to measure the edge in networks, so that the full properties of matrix algebra may then be employed as an analytical aid in better understanding business networks. There is a clear need in order to achieve this aim to utilise a more extensive set of data over a longer time period. In a sense this paper reaffirms that perhaps the most important aspect of network theory research is the space between entities, which is an idea many IMP researchers have held for a number of years.

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